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Anthony T. Ferrara

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LATERAL BRACING REQUIREMENTS OF PLASTICALLY DESIGNED BEAMS

by

Anthony T. Ferrara

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Master of Science

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May 22, 1962
(Date)

Theodore V. Galambos
(Professor in Charge)

Prof. W. J. Eney
(Head of Department)

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ABSTRACT

Results of a series of experiments that were performed to study the lateral bracing requirements of wide-flange beams are presented. The main reasons for this research were (1) the determination of the required bracing for beams subjected to constant plastic moment, (2) to study the effects of various methods of connecting the bracing members to the main beams, and (3) to study the effect of beam size on the rotation capacity of beams.

It was found that bracing with l/d ratios of less than approximately 30 seemed to be adequate and that the method of attachment made little difference.

1. INTRODUCTION

The problem of lateral bracing is of considerable importance in the plastic design of steel structures. One basic assumption in this method is that at certain points in a structure plastic hinges will occur and these must be capable of rotating through finite angles in order that the structure becomes a mechanism. The purpose of lateral bracing is to insure that hinges will rotate sufficiently to let a mechanism form. To accomplish this the bracing must firstly be spaced at proper intervals, and secondly, be strong enough to prevent members from lateral movement and axial twisting at the bracing points. The compression flanges only need be braced since they are critical with respect to lateral stability. It should be noted that the required rotation capacity (the angle of the hinge must rotate through) for each hinge in the structure is not the same but varies due to the fact that the hinges usually form one at a time and not simultaneously.

From tests performed at Lehigh University by Lee and Galambos[1], using the criterion that a rotation capacity of approximately 10 times the rotation at the theoretical inception of yielding was sufficient, it was found that the maximum permissible unbraced length of a rolled steel section subjected to a constant bending moment was about forty five times the weak axis radius gyration. If on the other hand, bracing was spaced a distance greater than 45 r_y the tests showed that the plastic moment (M_p) was still reached but was not maintained until the necessary inelastic rotation had occurred. It should be noted that since the results are for uniform moment (the most severe case) this 45 r_y is a lower, or safe limit, of bracing spacing. A theoretical "upper bound" solution to this problem was performed by Lee(2). As a result of this analysis the spacing of lateral bracing was found to be 40 r_y which is very close to the value obtained experimentally.

With the problem of spacing solved experimentally it was then desired to find the necessary strength and slenderness of the lateral bracing in order to insure the formation of plastic hinges of adequate rotation capacity. The scope of the investigation may best be illustrated by Fig. 1; shown in this figure are the two characteristic types of moment-versus-end rotation curves obtained for the laterally braced beams tested which all were subjected to uniform bending moment. The curve labelled "adequate" contains a "hinge plateau", and thus shows the formation of a plastic hinge. This type of curve was obtained if the bracing was strong enough to allow the beam to develop its full "post-buckling strength"(1). It can be seen that the beam is elastic (curve is linear) up to A; and at B it becomes fully "plastic" or yielded still sustaining the load while continuing to deform; and then finally, the compression flange of the beam buckles locally at point C and the load drops indicating failure. It should be noted that in few tests the beams even sustained load after local buckling of the compression flange, however, for the sake of comparison failure for all specimens was defined as occurring at the point of local buckling unless of course the load dropped off prior to this occurrence. The curve labelled "inadequate" (dotted) on the other hand, was obtained if the bracing was either too weak or too slender thereby causing the beam to unload before it developed a plastic hinge. As can be seen from the curves, the bahavior of all the beams tested was the same up to point B. It should be noted that lateral buckling of the beam cross section occurred in all the experiments regardless of the type of curve that resulted and usually started at about point B.

Some other tests were performed in addition to those just mentioned. In these experiments it was desired to investigate the effects of such incidentals as beam size, local buckling, method of beam to purlin (bracing member) attachment, half stiffeners in vicinity of plastic hinge, and bracing on one side only on the rotation capacity of plastically designed beams. It was observed that the moment-versus-end rotation curves for these tests took shapes similar to those in Fig. 1. Consequently then, the same criterion for failure which was discussed in connection with the experiments on purlin size and strength was used.

To date only a limited number of investigations have considered the problem of lateral bracing requirements. In papers by both Winter[3] and Zuk[4] of Cornell University it was concluded that in conventional (elastic) design the lateral bracing forces are quite small and consequently the problem of strength of lateral bracing is not critical in structural design. However, this conclusion, as will be obvious later, is not valid for plastic design since the deformations which the structural members must sustain at failure are considerably larger than during elastic behavior Also, in plastic design stiffnesses of members are greatly reduced due to the yielding that necessarily takes place.

This thesis considers only the experimental work discussed previously and does not include any theoretical solution to the problem. One can readily realize that a rigorous theoretical analysis of this problem of bracing requirements would be highly complicated because of the many variables involved (see Ref. 2, page 62). The prime purpose of this thesis is to establish, from a series of experiments, some criteria for the design of bracing that can be used until a theoretical solution is realized. The data presented herein should be of great value to the one who finally develops the theoretical solution.

2. DESCRIPTION OF THE EXPERIMENTAL PROGRAM

2.1 Scope of the Investigation

A series of fourteen tests were performed in this experimental program on the lateral bracing requirements of plastically designed beams. The loading conditions on the beams is shown schematically in Fig. 2. The center or critical span was subjected to a constant bending moment and was the span under consideration. In all the tests the beam specimens were divided into three equal spans except for test P-4 in which the adjacent spans were longer. All the beams had full stiffeners at each of the loading points (A, B, C, and D on Fig. 2) except in test P-9 where there were only half stiffeners at points B and C. Lateral support for each beam was provided at the loading points. At points A and D (Fig. 2) the lateral support was in the form of two knife-edge plates which prevented the beam from twisting axially and deflecting laterally but allowed rotations in both principal directions. The lateral support at points B and C was provided by I - shaped purlins which were attached to the compression flange of the test specimen.

2.2 Description of the Test Set-Up

2.2.1 Loading and Supporting System

A complete schematic diagram of the test set-up is shown in Fig. 3. The loading and supporting system was designed to provide simple support conditions for the test beam in the plane of loading. Equal loads were applied downward at the ends of the beam by a set of 55 kip hydraulic jacks* which were on a parallel pressure circuit of an Amsler pendulum dynomometer and controlled by a single valve. To simulate a knife-edge loading condition the load was transmitted to the test beam by a 2" roller.

Vertical supports in the form of two high strength steel rods were provided at the ends of the critical span of the test beam. These rods were pin-connected to a supporting girder above. This supporting girder was bolted at its ends to the center of two parallel rectangular frames, both of which were fixed to the laboratory floor. The rods were also pin-connected to the test beam either through two plates welded to the tension flange or through a yoke which transmitted the load to the compression flange. In the latter method the load was transmitted through the yoke by means of four, high-strength steel rods to a thick plate on the compression side of the beam. The load was applied to the beam by means of a roller placed between the plate and

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* 22 kip jacks were used for P-3 and P-4.

the beam in such a manner that the beam cross-section was free to twist. These two methods of connection will be referred to respectively as tension and compression flange loading and are shown in Fig. 4 and Fig. 5 (see Fig. 2 also)

2.2.2 Lateral Support Conditions

The lateral supports at the ends of the test beam (points A and D in Fig. 2) each consisted of two 1/2" plates bolted to the webs of a 5" channels which were in turn welded to a base plate. To create a knife-edge condition 1/2" diameter rods were welded to the edges of the plates and before each test, grease was applied along the length of the rods to keep friction to a minimum. There were slotted holes in the channels so that the distance between the plates could be adjusted. Both the sets of lateral supports were connected to a heavy base beam which was anchored to the laboratory floor. The specimen could deflect vertically between the knife-edges and rotate about both principal axes but was prevented from deflecting laterally or twisting axially.

Each specimen was laterally supported at the ends of the critical span by purlins connected to the compression flange. The purlins were pin-connected at their far ends to rigidly held cross beams. This allowed vertical rotation but prevented twisting and horizontal rotation of the purlin (see Fig. 3).

2.2.3 Instrumentation

During each test, deflections and curvatures in the loading and lateral direction were measured. In the elastic range, readings were taken for convenient increments of load while in the inelastic range, increments of deformation were used. Loading was stopped each time readings were taken. Furthermore, in the inelastic range, the readings were not taken until sufficient time had elapsed to permit the system to come to rest. Thus, the effects of rate of loading do not influence the results.

Vertical deflections of the test specimen were measured by means of a precise surveyor's level and a travelling 1/100 in. scale that was held vertically at each of thirteen points laid out across the entire length of the beam. Lateral deflections were measured using a transit fixed in a vertical plane and a 1/100 in. travelling scale. Lateral deflections of both the compression and tension flange were recorded.

Curvature of the test beam were obtained from strain readings and section geometry. Strains were obtained from pairs of SR-4 gages placed as shown in Fig. 6. The gages at the flange tips were used to determine the point of lateral buckling since at lateral buckling there was a significant difference in strain on the opposite tips of the compression flange. At each end of the critical span slope-measuring devices were used. These devices are shown schematically in Fig. 7 and can also be seen in Figs. 4 and 5. As can be seen from the schematic drawing 15" steel rods were connected to the stiffeners and thus rotated with the test beam. The amount of rotation was measured by two Ames Dials connected to the ends of the rods by thin steel wire. This gave slopes at each end of the critical span and since there was theoretically a constant moment across the span the curvature was obtained by dividing the change in slope by the length. The values of curvature so obtained were used as a check on those calculated using the strain readings and the two compared favorably.

An overall view of the test set-up is shown in Fig. 8. In this picture, transverse deflections are being measured.

2.3 Material Properties

All the beam specimens were rolled sections of ASTM A-7 structural steel and were produced from three different rollings (two for the 10WF25 section and one for the 8B 13 section). Three series of coupon tests were performed to determine the material properties of the steel from each rolling. Coupons were cut from the webs and flanges of the sections and from each series of coupon tests a weighted average yield stress was computed. The weight given to the yield stress values from the web and flange coupons were in the same proportion as the web and flange areas. The modulus of elasticity for the steel of each rolling was also obtained from the coupon tests. It should be noted here that coupons were tested at a low strain rate (.04 in. per minute) and that yield stresses obtained were static values[5].

In addition to the coupon tests, three short beam tests (length = 20 r_y) were performed to determine the value of the full plastic moment experimentally. Another yield stress estimate was obtained by dividing the full plastic moment of each short beam by its measured plastic modulus (Z). The yield stresses so obtained compared favorably with those from the coupon tests. (see table 1)

The full plastic moment for each beam specimen was determined by multiplying the yield stress (from coupon tests) and the measured plastic modulus of the respective beam. A summary of the material properties appears in Table 1.

The sectional properties of all the beam and purlin sections used in the entire experimental program appears in Table 2. The values in this table are nominal and not obtained from measurements.

2.4 Scope of the Experiments

The objectives of the first six tests (LB-12, 13, 14, 18, 19, 20) were to find the necessary requirements of strength and slenderness for the lateral bracing of beams. In all these tests the beam sections were 10WF25. The purlins used were continuous and welded with 1/4" fillets to the compression flange of the beam (see Fig. 9a). In LB-12, 13, and 14 the bracing sizes were varied (4I7.7, 3I5.7, and M2362 respectively) while their lengths were held constant. In these three tests the tension flange method of loading was used. It was found that the 3I5.7 section was the most suitable size for bracing therefore, in LB-18, 19 and 20 this section was used and the lengths were varied (l/d of purlins was 28, 49.3, and 38.7 respectively). The compression flange method of loading was used for these three tests. From these tests it was found that an 84" length of purlin was needed in order that a plastic hinge might form in the beam. As can be seen from Table 3, which gives a complete summary of the entire experimental program, tests LB-18 and LB-13 were exactly the same except for the type of loading. This was done to determine whether or not the method of loading affected the test results in any way.

Test LB-22 was designed to furnish information about beams with bracing on one side only. A condition such as this would occur in an end bay of a multibay building. The 10WF25 beam section was used with 3I5.7 purlins welded to the compression flange. To allow for the absence of purlins on one side of the beam, the length of the remaining purlins was decreased by one-third to 56".

The purpose of test P-3 was to determine the effects of beam size and local buckling on rotation capacity. An 8B 13 section was specifically chosen for the beam specimen as it has a more critical $^b/t$ ratio[7] that the 10WF25 (see Table 2). The purlins were M2362 sections and were again welded to the compression flange of the beam. The length-to-depth ratio of the purlins was 28, a value which proved to be sufficient in both tests LB-13 and LB-18.

In test P-4 the adjacent span lengths were increased from the usual $40r_y$ to $60r_y$ in order to determine if this would have any effect on the rotation capacity of the beam.

In order to investigate the effects of different beam-to-purlin connections tests P-6, P-7, P-8, and P-10 were included in the test program. The purlins for all these tests were 3I5.7 sections. In tests P-6, P-7, and P-8 the beams were braced on both sides by 84", 3I5.7 section purlins. In test P-10 the purlins were on one side only and were 56" long. The purlins in test P-6 were cut at their point of connection on the compression flange of the beam leaving about a 1" gap (see Fig. 9b), whereas in previous tests they had been continuous across the beam. In this test the purlins were welded to the beam. In P-7 the purlins were again continuous but were connected to the beam with 3/8" machined bolts (see Fig. 9c). There were four bolts at each support point. Test P-8 was the same as P-6 in that the purlinswere discontinuous. However, in P-8 bolts were used to connect the purlins to the beam (see Fig. 9d). For test P-10 the purlins were on one side only and bolted to the compression flange of the beam specimen.

In test P-9 the usual full stiffeners at the ends of the critical span were replaced by half stiffeners extending from the compression flange to the middle of the web. The arrangement can be seen in Fig. 5. The purlins in this test were continuous and welded to the beam.

To summarize it might be said that the tests fell into essentially five categories which are (1) determination of purlin size, (2) determination of purlin strength, (3) determination of effect of beam size, (4) determination of effect of purlin on one side only, and (5) determination of effect of beam-to-purlin attachment.

3. DISCUSSION OF THE EXPERIMENTAL RESULTS

3.1 Moment versus Curvature Relationships

The principal results obtained from this experimental program are presented in the form of moment versus curvature (M-ϕ) curves. The moment in the critical span corresponding to a specific endload P, non-dimensionalized by M_p, is the ordinate, and the measured curvature ϕ at the center of the critical span, corresponding to the same load P and non-dimensionalized by ϕ_y, the curvature at the theoretical start of yielding, is the absissa. The values of curvature were computed from the strain gage (see Fig. 6) readings at the center of the critical span. The value of ϕ/ϕ_y plotted in each case represents the average of three strain gage readings.

The moment versus curvature relationships for the entire test program appear in Figs. 12 through 16 with the points of lateral and local buckling noted. To give a clear picture of these buckling phenomena Figs. 10 and 11 are included. Fig. 10 shows a typical beam after its compression flange has buckled laterally. It may be pointed out here that the top flange (tension flange) did not move however, this will be substantiated later. A typical local buckling failure is shown in Fig. 11.

Fig. 12 shows the M-ϕ curves for the tests which were performed to determine the necessary strength of lateral bracing. It can be seen that the performance of the specimens in both test LB-12 (purlin length = 21d) and LB-13 (purlin length = 28d) was adequate in that the plastic moment was reached and also maintained until sufficient rotation had occurred. It should be noted that in each case the beams continued to rotate after lateral buckling without any decrease in load. The M-ϕ curve for LB-14 (purlin length = 32d) on the other hand is of a different nature in that after it reached its ultimate moment it immediately began to unload (gradually at first) and continued this unloading until the completion of the test. In some later tests the beams unloaded but afterwards showed an increase in load before final unloading. The performance of the specimen in test LB-14 was considered to be inadequate in view of the fact that it did not sustain the required moment (M_p) for the required rotation. The point of local buckling of the compression flange is shown on the curves of LB-13 and LB-14 but in test LB-12 local buckling took place at a curvature of 20.8 and therefore could not be shown on this graph. It should be noted here that the speci men of test LB-13 acted differently than most others in the program in that it sustained the required load even after local buckling of the compression flange.

The M-ϕ curves for tests LB-18, LB-19, and LB-20 which were performed to determine purlin slenderness, are shown in Fig. 13 with the curve from test LB-13 included for comparison. In this group only specimen LB-18 underwent sufficient rotation at load M_p and was considered adequate. In both test LB-19 and LB-20 the plastic moment was reached but was not maintained. It can be seen from an examination of the curve for LB-18 that the major portion of the rotation capacity of the beam lies in the post-buckling range (that is: a large amount of rotation occurs after the beam has buckled laterally) and also that local buckling seems to cause the load to drop off.

The effect of tension flange and compression flange loading can be compared by examining the M-ϕ curves for LB-13 (tension flange loading) and LB-18 (compression flange loading) in Fig. 13. Both curves have the same general shape but the curve for the beam with tension flange loading is somewhat higher. This fact would seem to justify what might be expected (that is that compression flange loading is more critical) however, the difference is so small (about 3%) that the two types of loading may be considered to have the same effect.

In Fig. 14 the curves from tests LB-22 and P-10 (purlins on one side only - in the former welded, in the latter bolted) are shown with the curve from LB-18 for comparison. It can be seen that even though the purlin was appreciably decreased in length, as was noted earlier, the performance of these specimens was still not adequate. In both tests the M_p of the beam was not reached (92% in LB-22 and 96% in P-10) and besides when a maximum load was reached it fell off before enough rotation had taken place. The fact that the moment attained in test LB-22 was less than P-10 may be attributed to a slight movement of the purlin supports during test LB-22. A possible reason why either of the beams did not reach M_p will be discussed subsequently.

The M-ϕ curves for the two tests (P-3 and P-4) using an 8B 13 section as the beam specimen are shown in Fig. 15. Both curves are well below the predicted value of M_p based on the coupon test results which may be an indication that beam size has an effect on the maximum moment reached. However, it should be noted that the yield stress obtained from the coupon tests was an extremely high 41.8 ksi (nominal 33 ksi) for this section. The yield stresses obtained for the 10WF25 section used for tests LB-22 through P-10 was also quite high (39.93 ksi). These high yield stresses made the M_p's of the beams high and consequently this might be the reason for the beams not reaching M_p. The fact that the maximum moment reached by the beam with longer adjacent spans was higher than the maximum reached by the beam with equal spans is not readily explainable. Since the two beams were exactly the same except for the adjacent span length, it would seem that P-4 ought to be weaker, because of the decrease of stiffness in the lateral direction offered by the adjacent span. However, exactly the opposite was observed. It seems that this can only be attributed to some local effect. From Fig. 13 it can be seen that the two curves, even though they do not reach M_p, reach a plateau at which the loads remain essentially constant and do rotate considerably a sufficient amount before unloading. It may be noted here that the values of maximum moment in these two tests, and, as a matter of fact, in all the others are well over the values obtained by using the nominal 33 ksi for σ_y. For example, if 33 ksi were used the maximum ordinate on the M- curve for test P-3 would be 1.16 instead of .913. (See Table 4) Also the rotation capacity would be increased if a lower value of σ_y were used since this would make ϕ_y smaller (i.e. ϕ/ϕ_y larger).

The final four M-ϕ curves are shown in Fig. 16. These curves represent the results of tests with different types of beam-to-purlin connections (P-6, P-7 and P-8) and of the test in which the beam had half stiffeners (P-9). The curves for P-7 (bolted continuous) and P-8 (bolted discontinuous) are similar in that they both reach a peak moment of about 95% M_p and then go through sufficient rotations until finally unloading at local buckling. The curve for test P-9 (half stiffeners) reaches approximately 95% M_p but local buckling takes place a little too early ($\phi/\phi_y = 8.12$) for it to be considered sufficient. The curve for test P-6 (welded discontinuous) reaches a peak moment of approximately 93% M_p but undergoes a sufficient amount of rotation at this level before local buckling.

Table 4 lists all the pertinent results from the entire experiment program. Results using the yield stresses computed from the short beam tests are also tabulated for the sake of comparison with those using yield values from the coupon tests (these were used in plotting the curves). Values computed from using the nominal 33 ksi yield stress are also listed.

3.2 Lateral Deflections

The lateral deflections at midspan for both flanges as a function of moment are plotted in Fig. 17 for two typical specimens (LB-19 and P-3). It can be seen that the tension flanges hardly moved while there was considerable movement of the compression flanges. In all the tests lateral buckling occurred in the inelastic range and the deflections increased with strain until local buckling of the compression flange. The lateral deflection of the compression flange at local buckling is recorded in Table 4 for each test, however, there seems to be no correlation between these values and the values of strain or curvature as might be expected. In Fig. 18 the laterally deflected shape of a typical beam is shown for several loads. It should be noted that in two tests (LB-13 and LB-20) the critical span buckled in double curvature but there seems to be no reason for this occurrence which, by the way, did not effect the behavior of the beams in any way.

3.3 Transverse Deflections

In Fig. 19 the moment is plotted against the vertical deflection at the center of the critical span for two typical tests. The vertical supports (tension rods) had elastic deformations as the load increased from zero to its maximum value therefore, the deflections shown in Fig. 19 have been adjusted to take this into account. These moment-deflection curves are similar to the M-ϕ curves previously discussed. The vertical deflection at the center and at the ends at local buckling for each test are included in Table 4. The deflected curves at various stages of loading are shown in Fig. 20. These are again from a typical test in that these curves were similar for all the tests. As may be seen from Fig. 20, lateral buckling took place rather early before large deflections had occurred which implies that the post-buckling strength is a major factor in the performance of the beams tested.

3.4 Comparison with Plate-Buckling Theory

In Table 4 the strains of the compression flange at local buckling are listed for each test. This value can be compared with values obtained from results of a theoretical solution of the plate buckling problem by G. Haaijer[6]. If it assumed that local buckling takes place at the onset of strain-hardening (a common assumption), this theory predicts, for 10WF25 and 8B 13 sections, that the strain will be about .0139 in/in. From an examination of the values obtained from the strain-gages it seems that this theory is a good lower-bound.

4. CONCLUSIONS

The following conclusions may be drawn from the results of the investigation presented in this paper.

1. A bracing member with a l/d ratio of about 30 or less may be considered to be sufficient. A plot of ϕ/ϕ_y at failure versus the l/d ratio of the purlins is shown in Fig. 21 to illustrate this point.

2. If a beam has bracing on one side only this bracing should have a l/d or somewhat less than 18 (in our tests l/d = 18.7).

3. The manner in which the purlins are attached to the beams seems to make little difference.

4. Half stiffeners in the vicinity of plastic hinges seem to be inadequate therefore full (fitted) stiffeners should be used.

5. It may be assumed that local buckling of the compression flange occurs at a strain approximately equal to the strain at the onset of strain-hardening.

5. NOMENCLATURE

M = Bending moment

M_p = Full plastic moment = $\sigma_y \cdot Z$

Z = Plastic modulus

L_{cr} = Length of span under consideration

L_{adj} = Length of adjacent span

l = Length of purlins

d = Overall depth of a section

b = Flange width

t = Flange thickness

w = Web thickness

r_y = Radius of gyration about the y-y axis

ϕ = Curvature

ϕ_y = Curvature at the start of yielding

σ_y = Yield stress

E = Modulus of elasticity

ksi = Kips per square inch

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7. TABLES AND FIGURES.

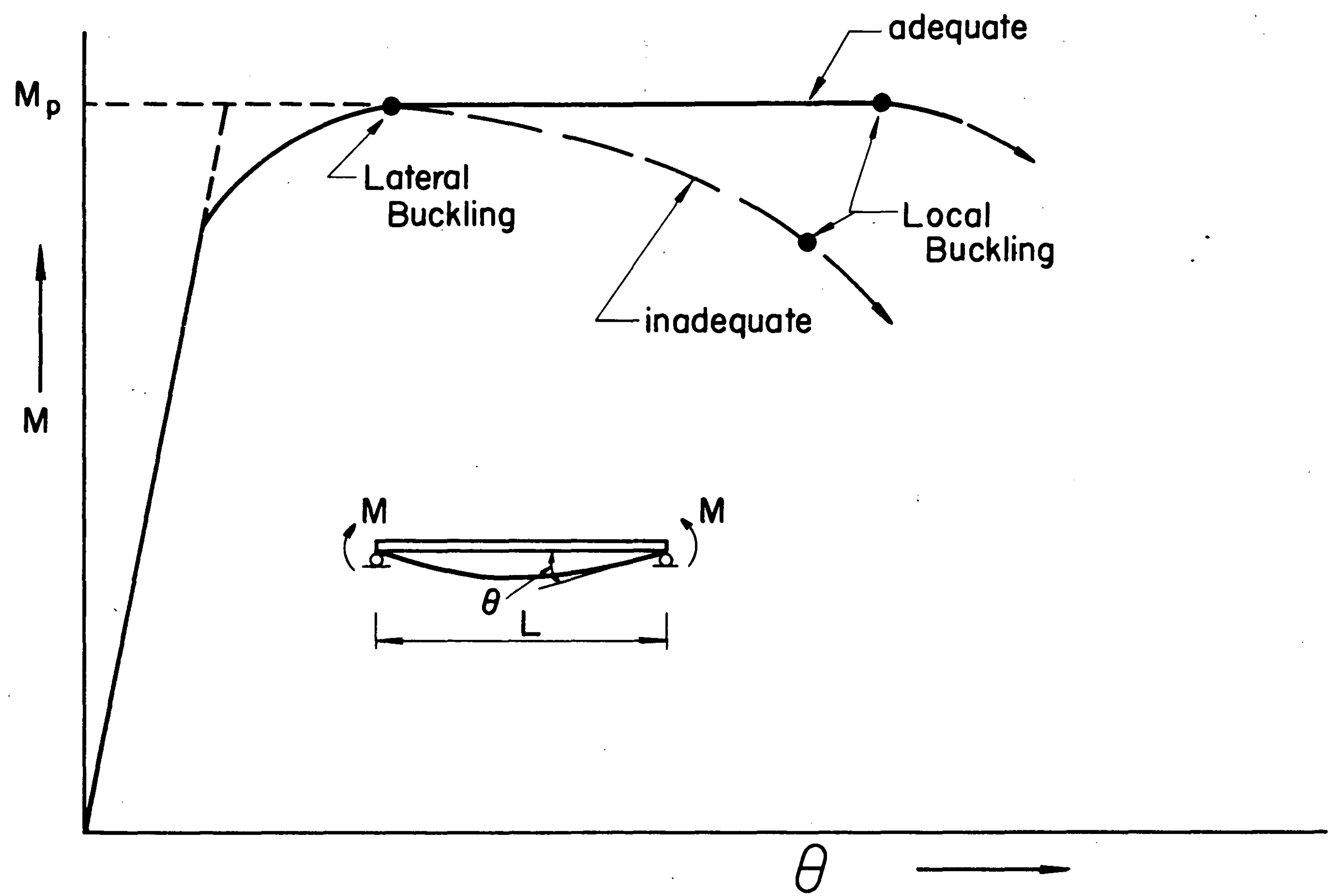
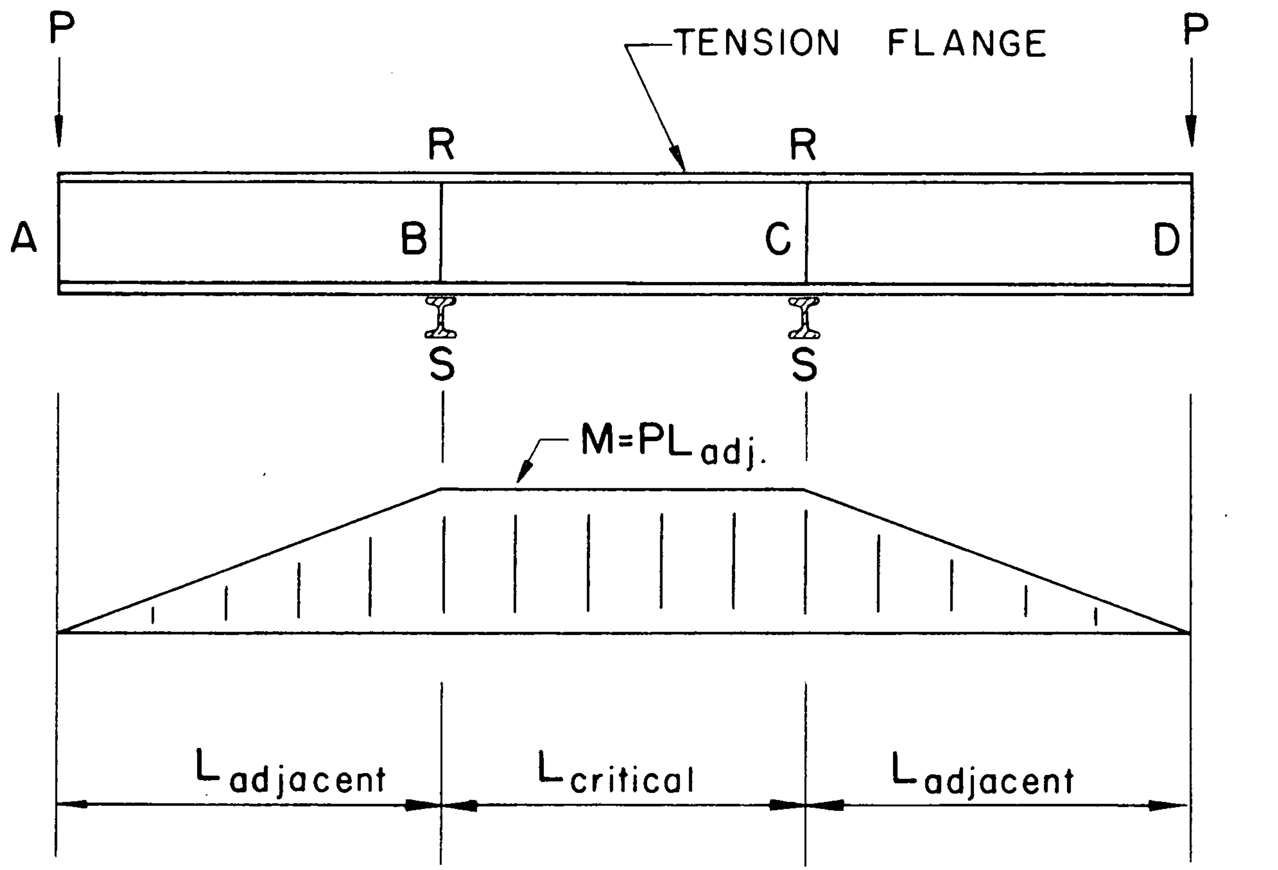


FIG. 1. SCHEMATIC DIAGRAM OF TYPICAL MOMENT ROTATION RELATIONSHIPS



R — POINTS OF LOADING FOR LB-12, 13, 14 AND P-6,8

S — POINTS OF LOADING FOR LB-18, 19, 20, 22 AND P-3,4,7,9,10

A, B, C, D, POSITIONS OF LATERAL SUPPORT

FIG. 2 SCHEMATIC DIAGRAM OF LOADING SYSTEM

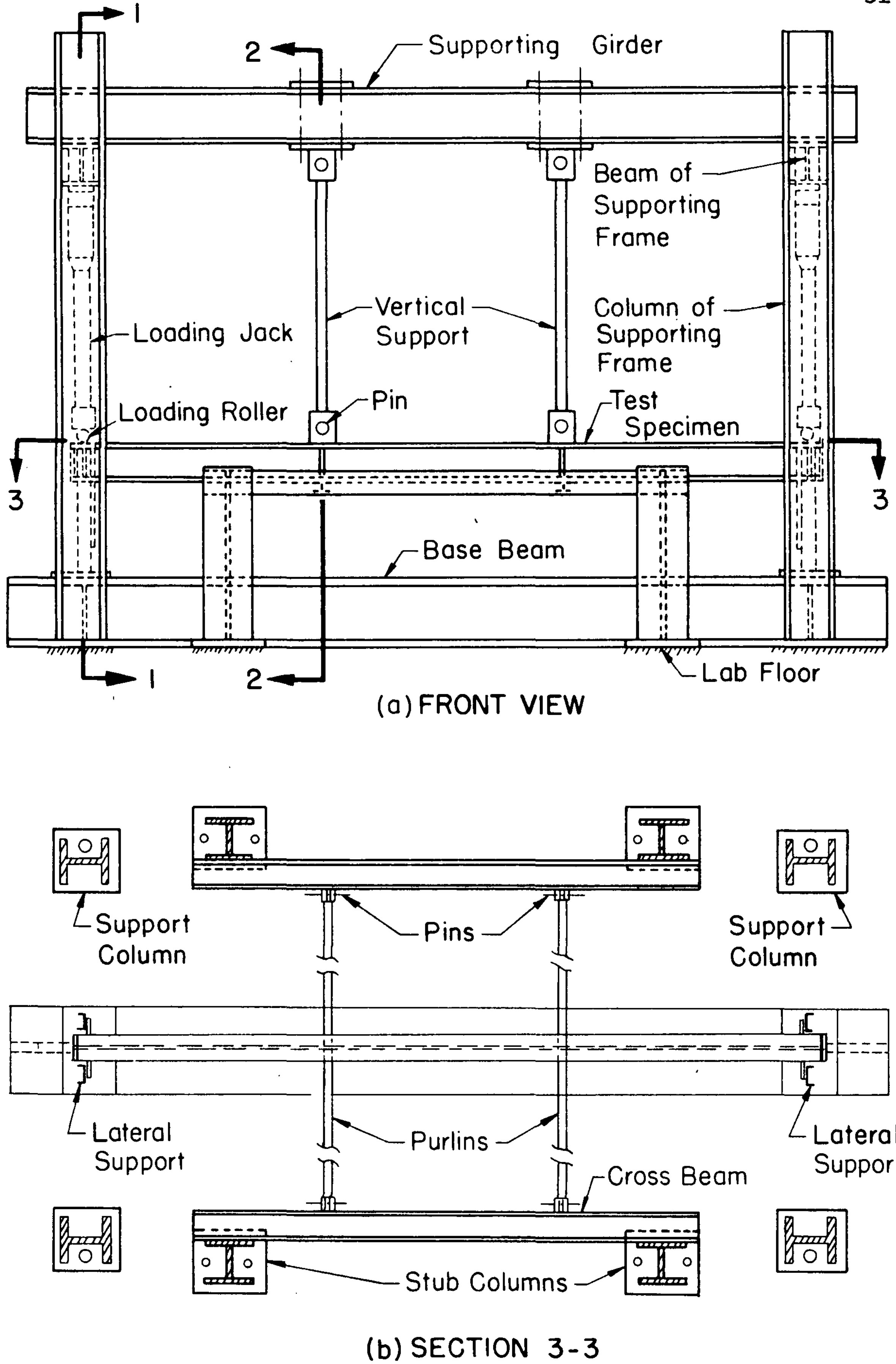
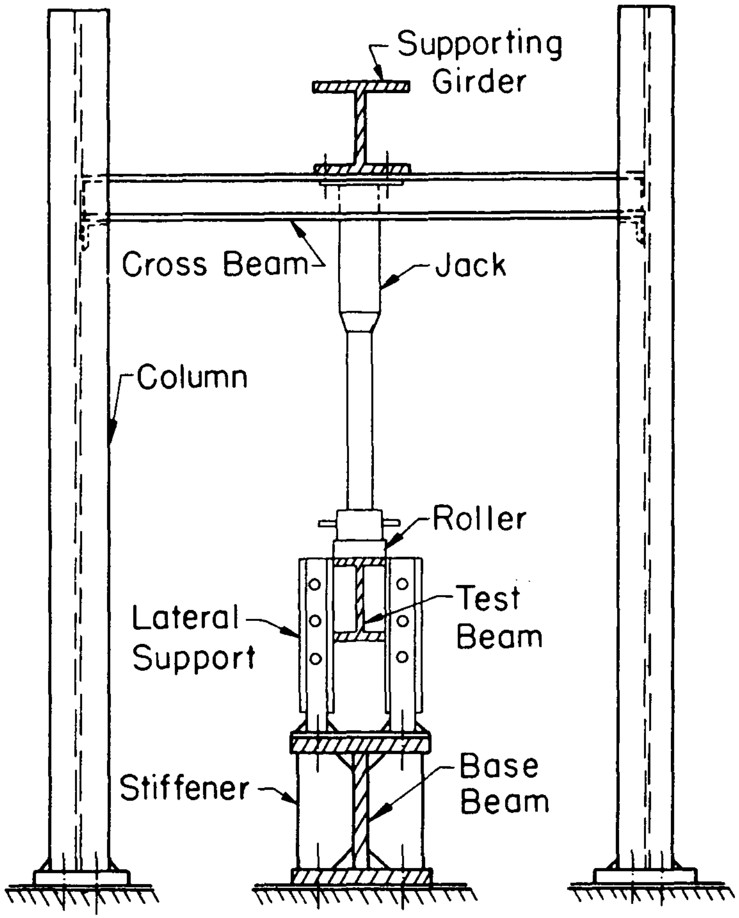
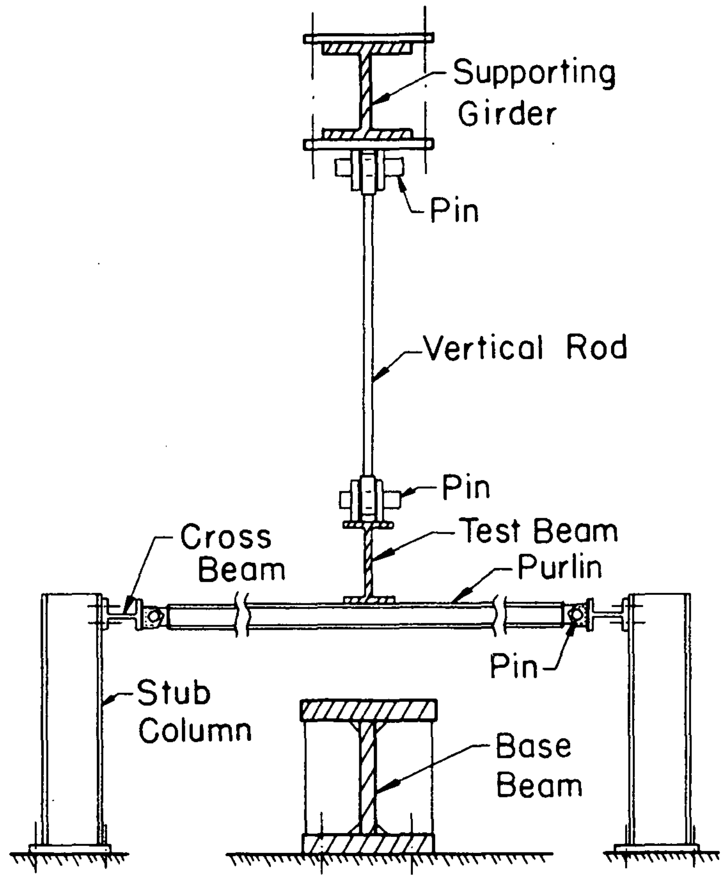


FIG. 3 SCHEMATIC SKETCH OF TEST SET-UP



(c) SECTION 1-1



(d) SECTION 2-2

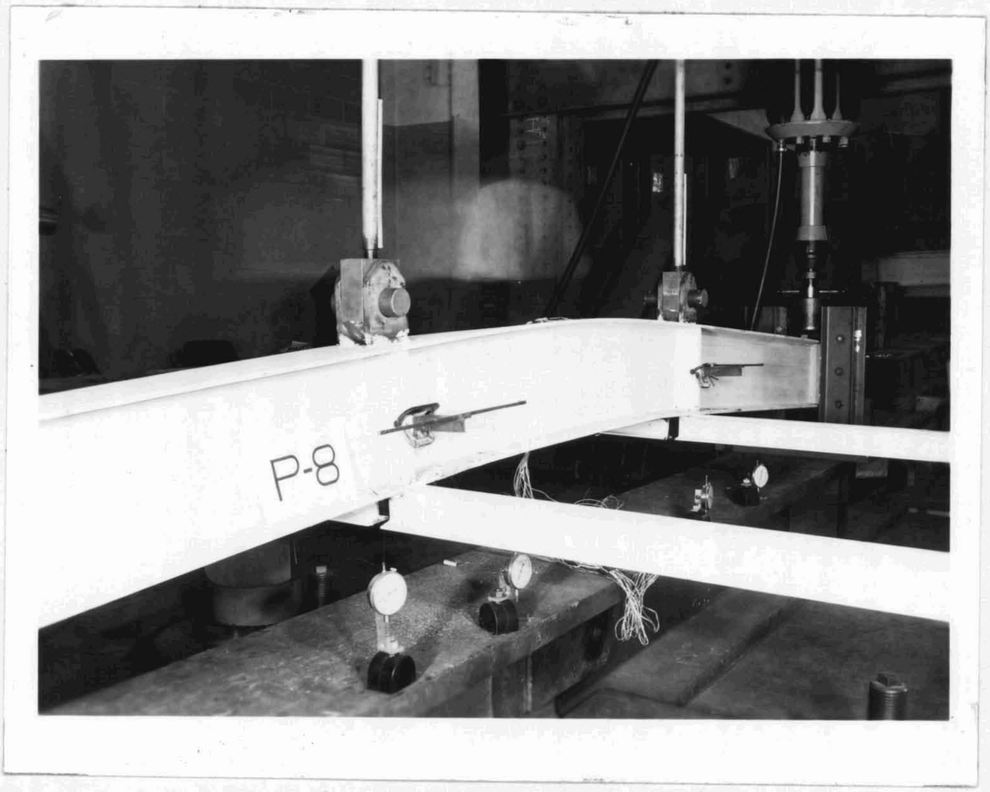


FIG. 4 TENSION FLANGE LOADING

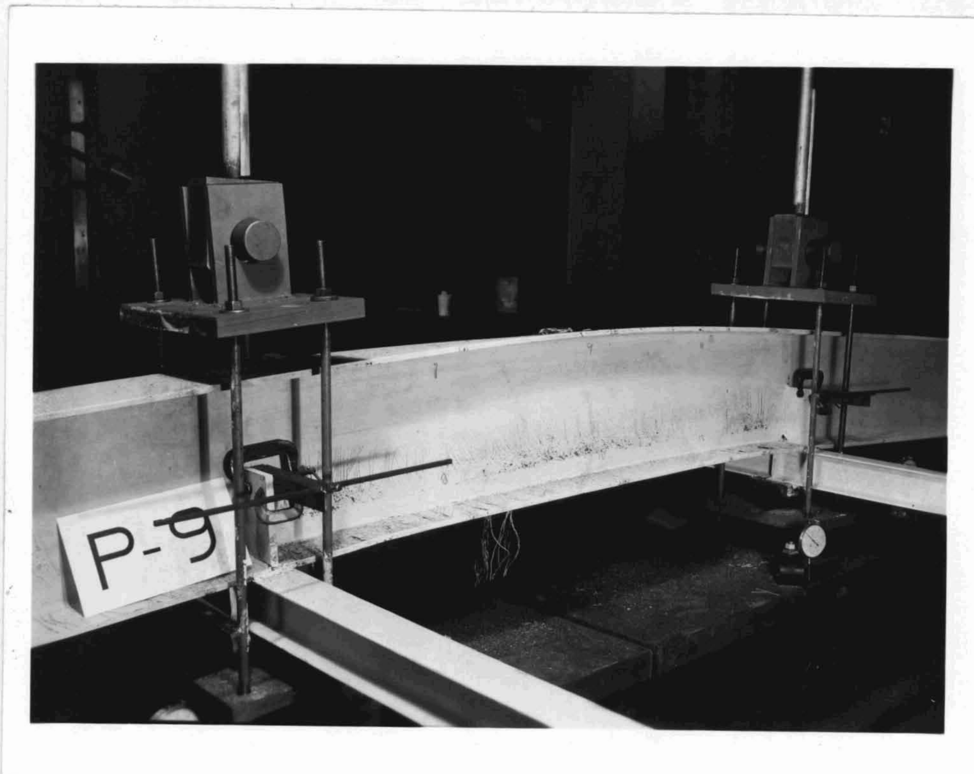


FIG. 5 COMPRESSION FLANGE LOADING

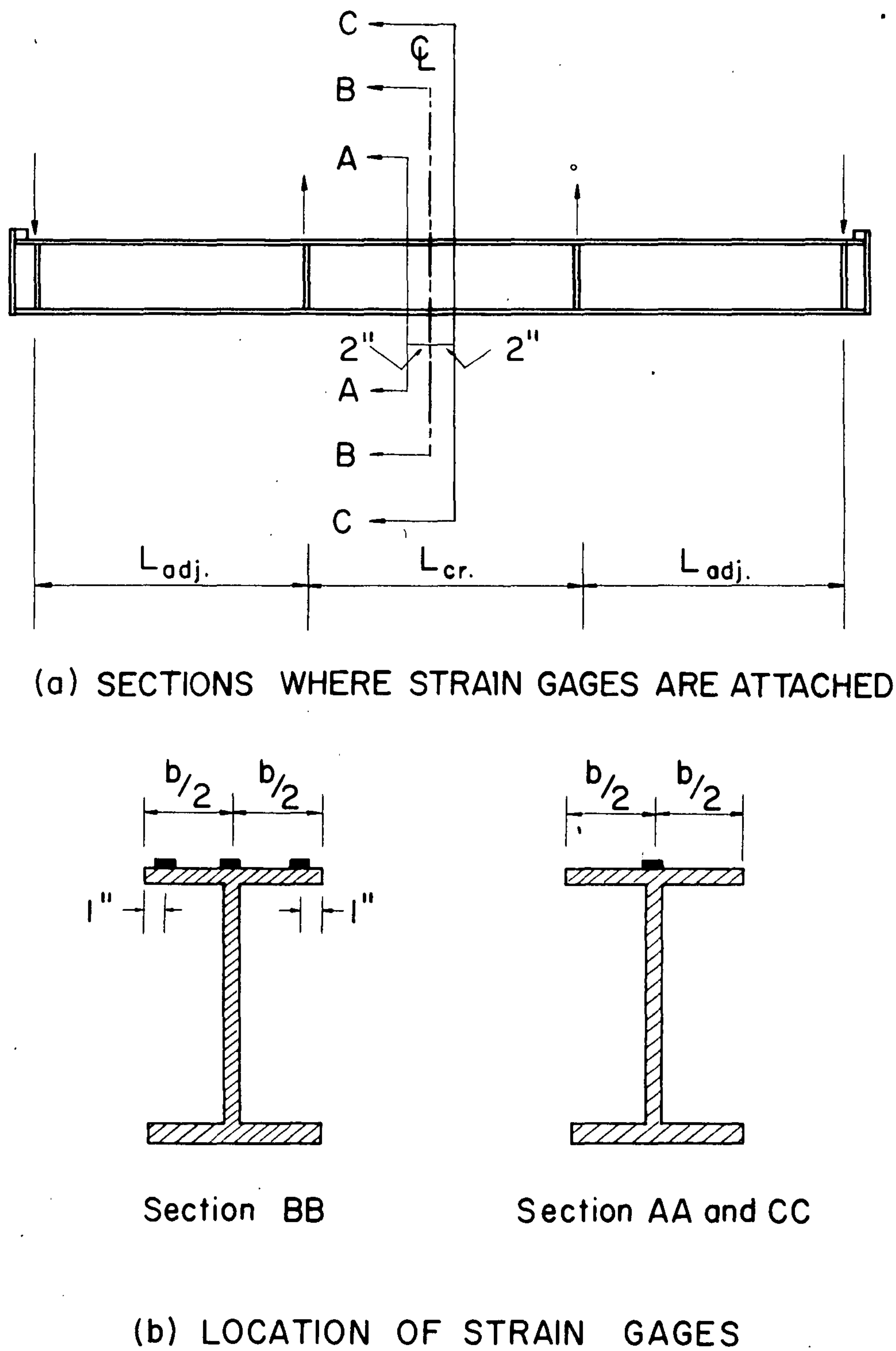


FIG. 6 ARRANGEMENT OF SR-4 STRAIN GAGES

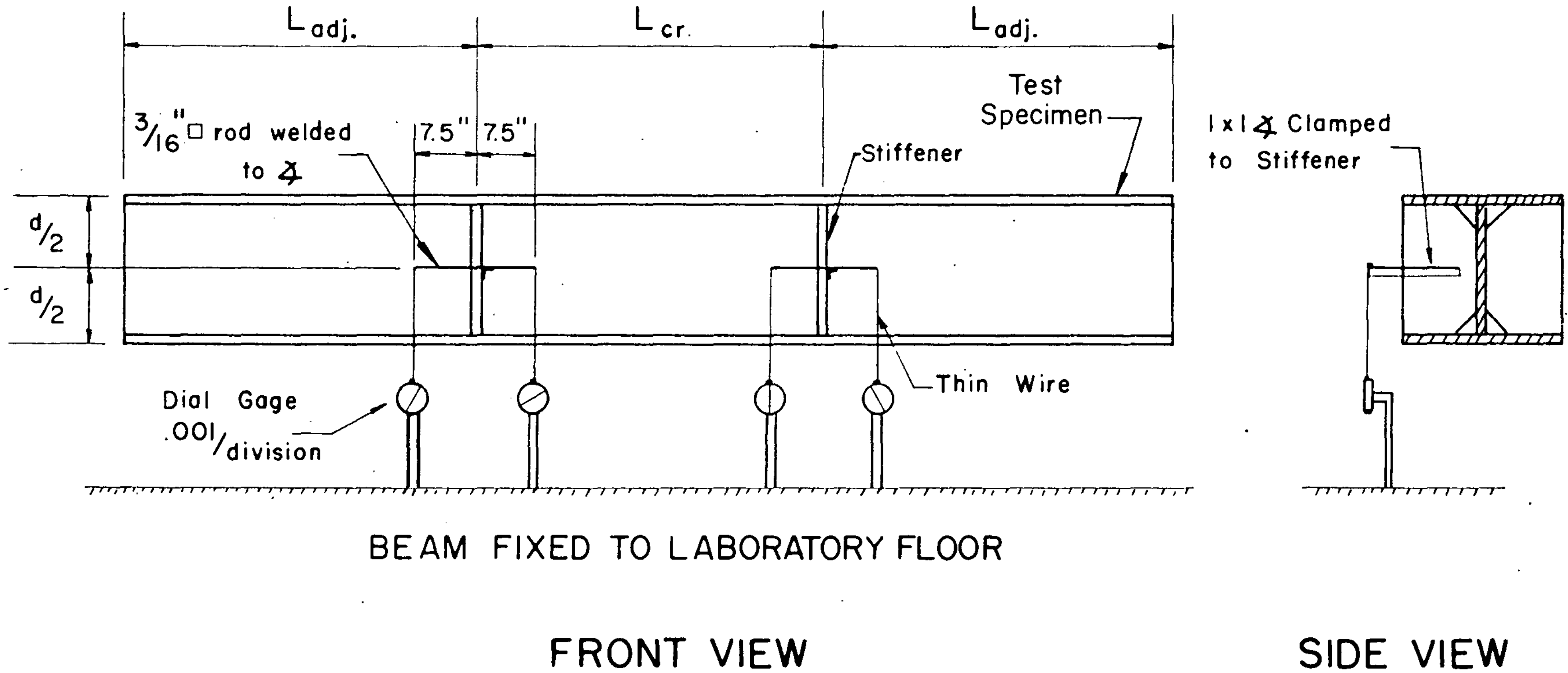


FIG. 7 SLOPE GAGE ARRANGEMENT

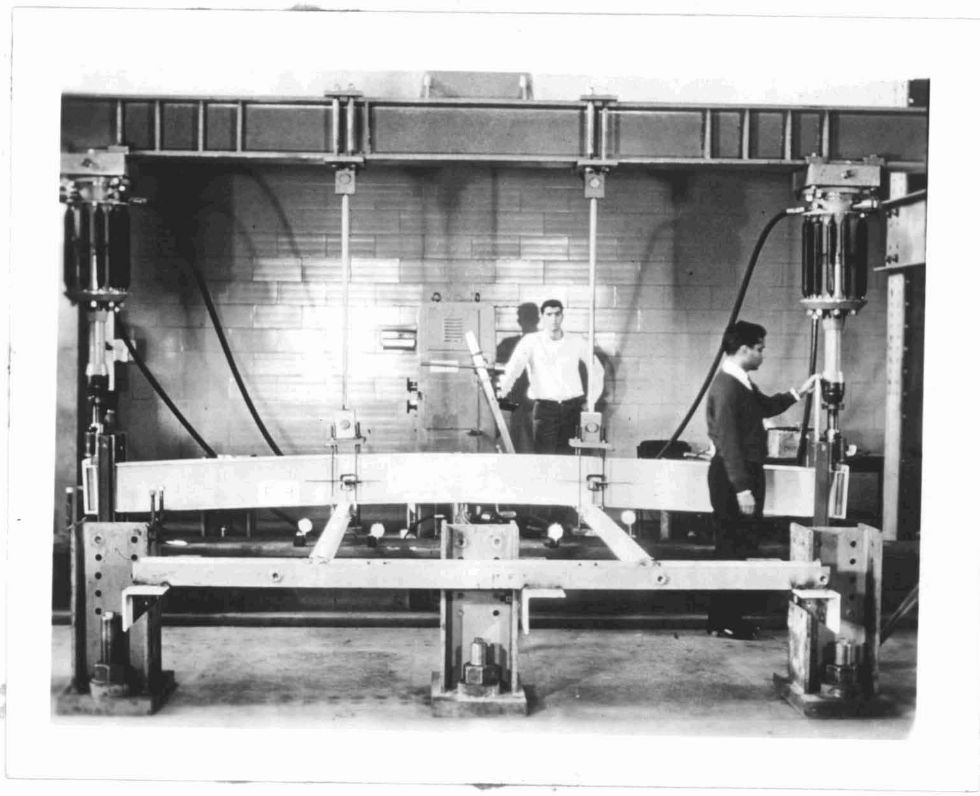
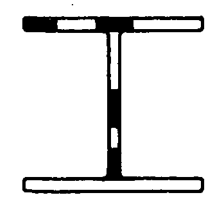


FIG. 8 TEST IN PROGRESS

ITEMS		10 WF 25						8B13		10 WF 25						NOMINAL
		LB-12	LB-13	LB-14	LB-18	LB-19	LB-20	P-3	P-4	LB-22	P-6	P-7	P-8	P-9	P-10	
TENSION COUPON DATA	σ (ksi)	σ_{yf} = 33.95 σ_{ya} = 35.5 σ_{yw} = 38.75  *						σ_{yf} = 38.5 σ_{ya} = 41.8 σ_{yw} = 47.65		σ_{yf} = 38.75 σ_{ya} = 39.93 σ_{yw} = 43.75						33
	E (ksi)	30,400						30,000		29,400						30,000
SHORT BEAM TEST RESULTS	M_p (k-in.)	1095						466		1145						980 (10WF25) 376 (8B13)
	σ_y (ksi)	37.5						40.8		37.6						33
PLASTIC MODULUS	Z (in.^{3})	28.59	29.05	28.75	28.86	28.6	29.0	12.0	11.76	30.4	29.2	29.2	29.1	29.5	29.5	29.5 (10WF25) 11.4 (8B13)
FULL PLASTIC MOMENT	$Z\sigma_{ya}$ (k-in.)	1013	1031	1020	1023	1014	1030	502	492	1212	1164	1164	1162	1178	1178	980 (10WF25) 376 (8B13)

*Position of coupons same for three rollings

TABLE 1 - SUMMARY OF MATERIAL PROPERTIES

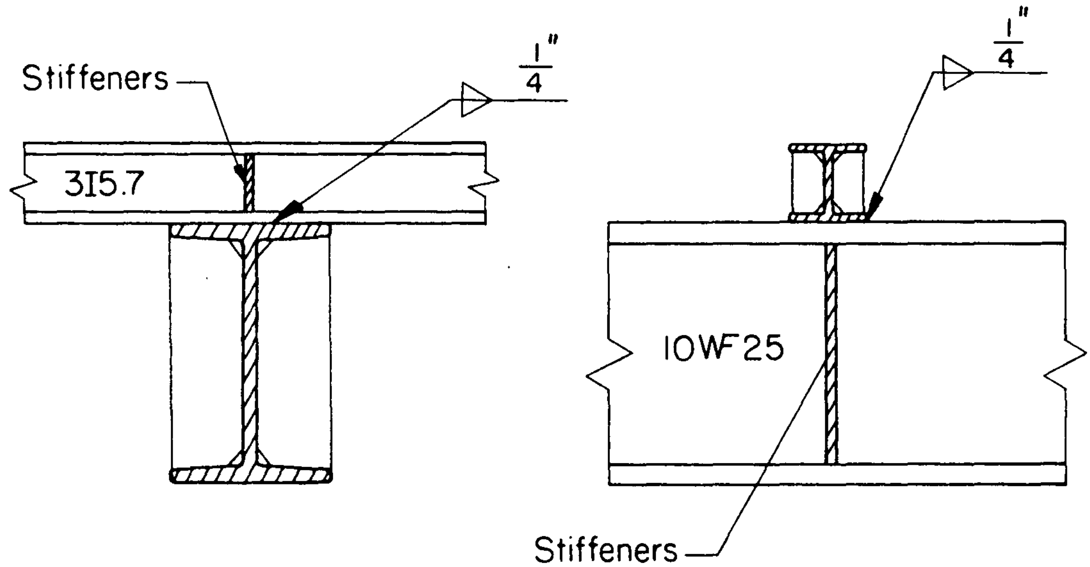
SECTION	AREA OF SECTION A, sq.in.	FLANGE WIDTH b, in.	DEPTH OF SECTION d, in.	WEB THICKNESS w, in.	FLANGE THICKNESS t, in.	$\frac{d}{w}$	$\frac{b}{t}$	MOMENT OF INERTIA about x-axis I_x, $in.^4$	RADIUS OF GYRATION about y-axis r_y, in.
10WF25	7.35	5.762	10.08	0.252	0.430	40.0	13.4	133.2	1.31
8B13	3.83	4.00	8.00	0.230	0.254	34.8	15.75	39.5	0.83
4I7.7	2.21	2.66	4.00	0.190	0.293	21.0	10.0	6.0	0.59
3I5.7	1.64	2.33	3.00	0.170	0.260	17.6	9.0	2.9	0.52
M2362*	1.085	1.840	2.625	0.156	0.201	16.8	9.2	1.236	0.407

*Special section produced by Bethlehem Steel Co.

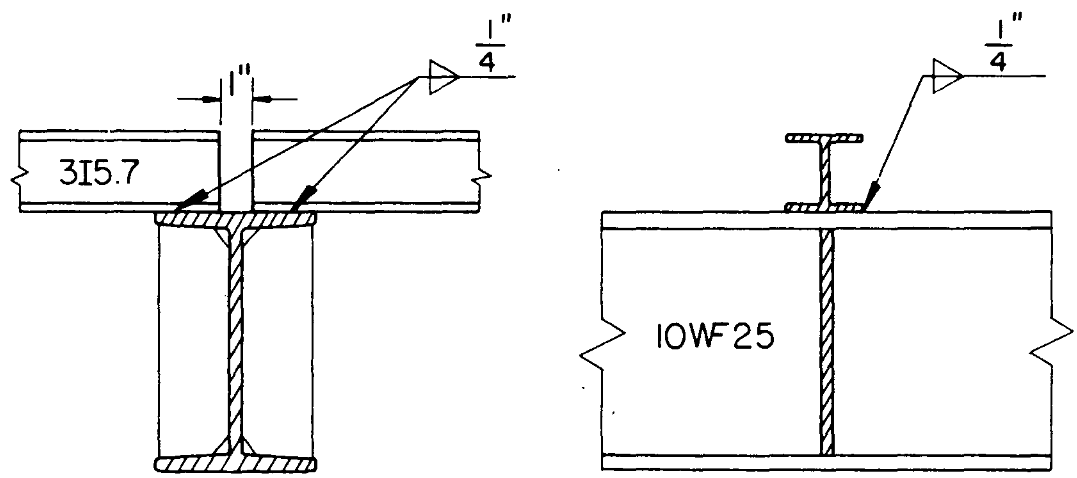
TABLE 2 - SUMMARY OF SECTIONAL PROPERTIES

TEST NO.	BEAM			PURLINS				PURPOSE
	SIZE	SPAN LENGTH CRITICAL	SPAN LENGTH ADJACENT	SIZE	LENGTH l/d	LENGTH l/r_y	LENGTH INCHES	
LB-12	10WF25	$40r_y$	$40r_y$	4I7.7	21	142	84	Effect of purlin size
LB-13	"	"	"	3I5.7	28	159	"	"
LB-14	"	"	"	M2362	32	206	"	"
LB-18	"	"	"	3I5.7	28	159	"	Effect of purlin slenderness
LB-19	"	"	"	"	49.3	279	148	"
LB-20	"	"	"	"	38.7	219	116	"
LB-22	"	"	"	"	18.7	105	56	Effect of purlins on one side only (welded)
P-3	8B13	"	"	M2362	28	180	73.5	Effect of beam size and local buckling
P-4	"	"	$60r_y$	"	"	"	"	Effect of length of adjacent spans
P-6	10WF25	"	$40r_y$	3I5.7		159	84	Effect of beam-to-purlin connection —— discontinuous purlins welded
P-7	"	"	"	"	"	"	"	Effect of beam-to-purlin connection —— continuous purlins bolted
P-8	"	"	"	"	"	"	"	Effect of beam-to-purlin connection —— discontinuous purlins bolted
P-9	"	"	"	"	"	"	"	Effect of half stiffeners
P-10	"	"	"	"	18.7	105	56	Effect of purlins on one side only (bolted)

TABLE 3 - SUMMARY OF TEST PROGRAM

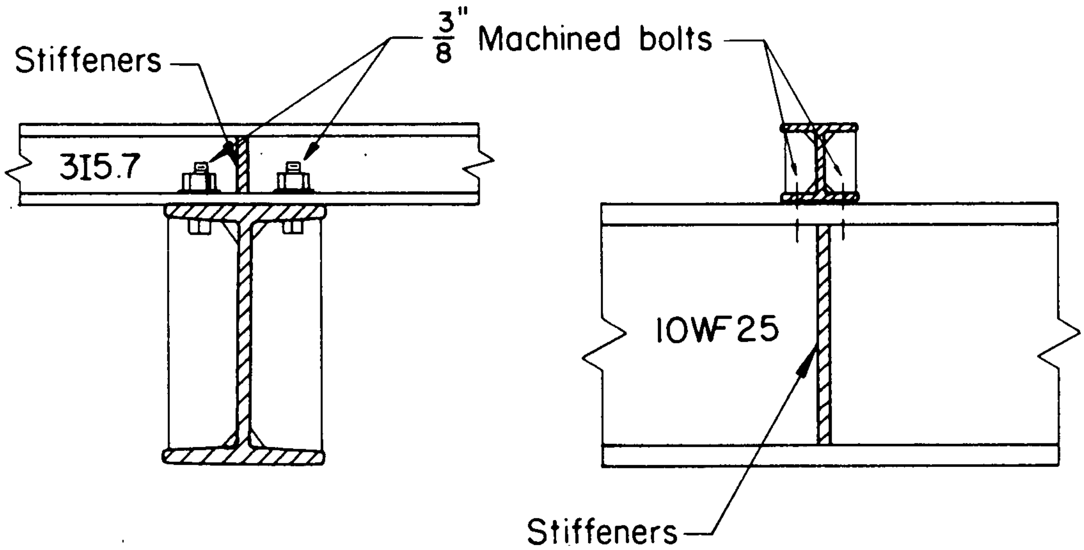


(a) WELDED CONTINUOUS CONNECTION (LB-18 & OTHERS)

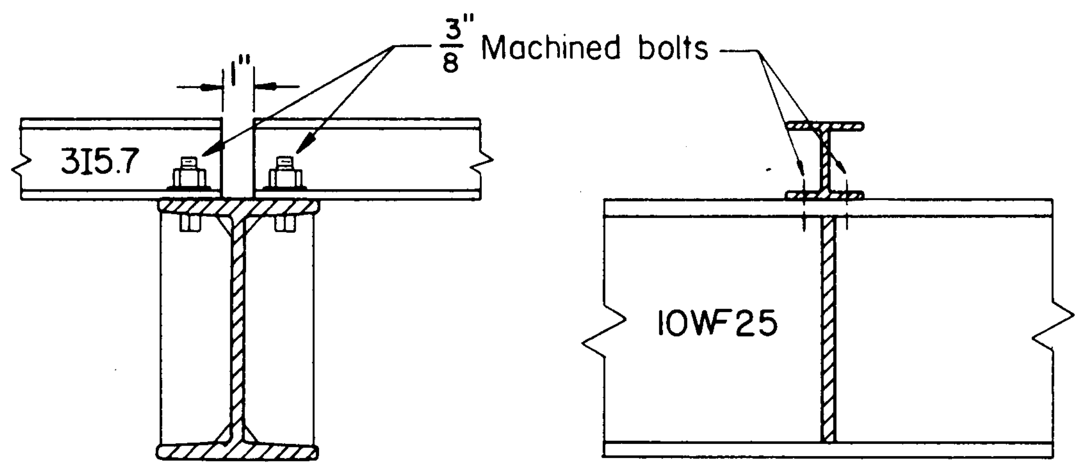


(b) WELDED DISCONTINUOUS CONNECTION (P-6)

FIG. 9 BEAM-TO-PURLIN CONNECTIONS



(c) BOLTED CONTINUOUS CONNECTION (P-7)



(d) BOLTED DISCONTINUOUS CONNECTION (P-8)

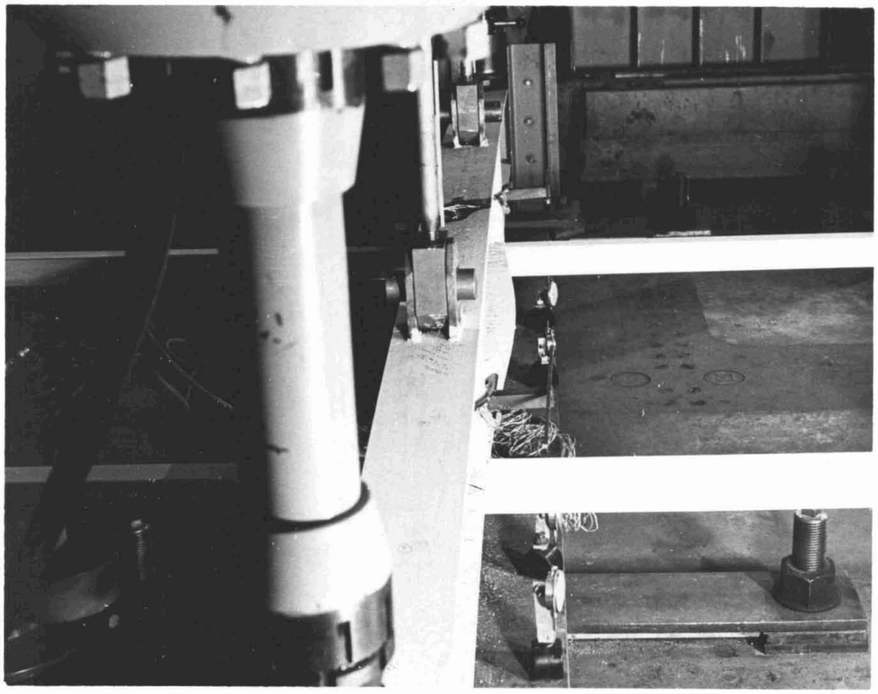


FIG. 10 LATERAL BUCKLING

FIG. 11 LOCAL BUCKLING

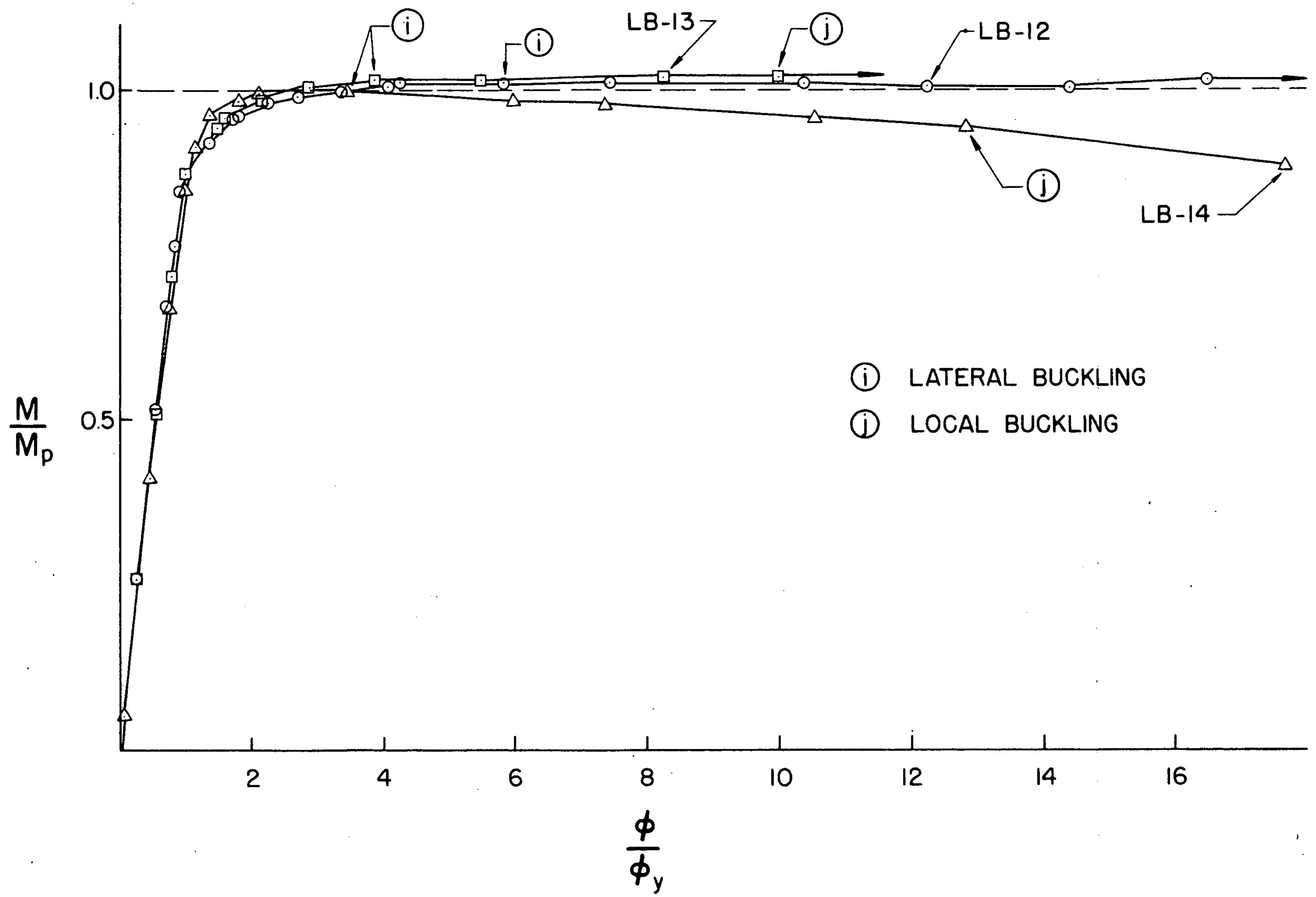


FIG. 12 MOMENT VERSUS MIDSPAN CURVATURE RELATIONSHIPS FOR LB-12, 13 AND 14

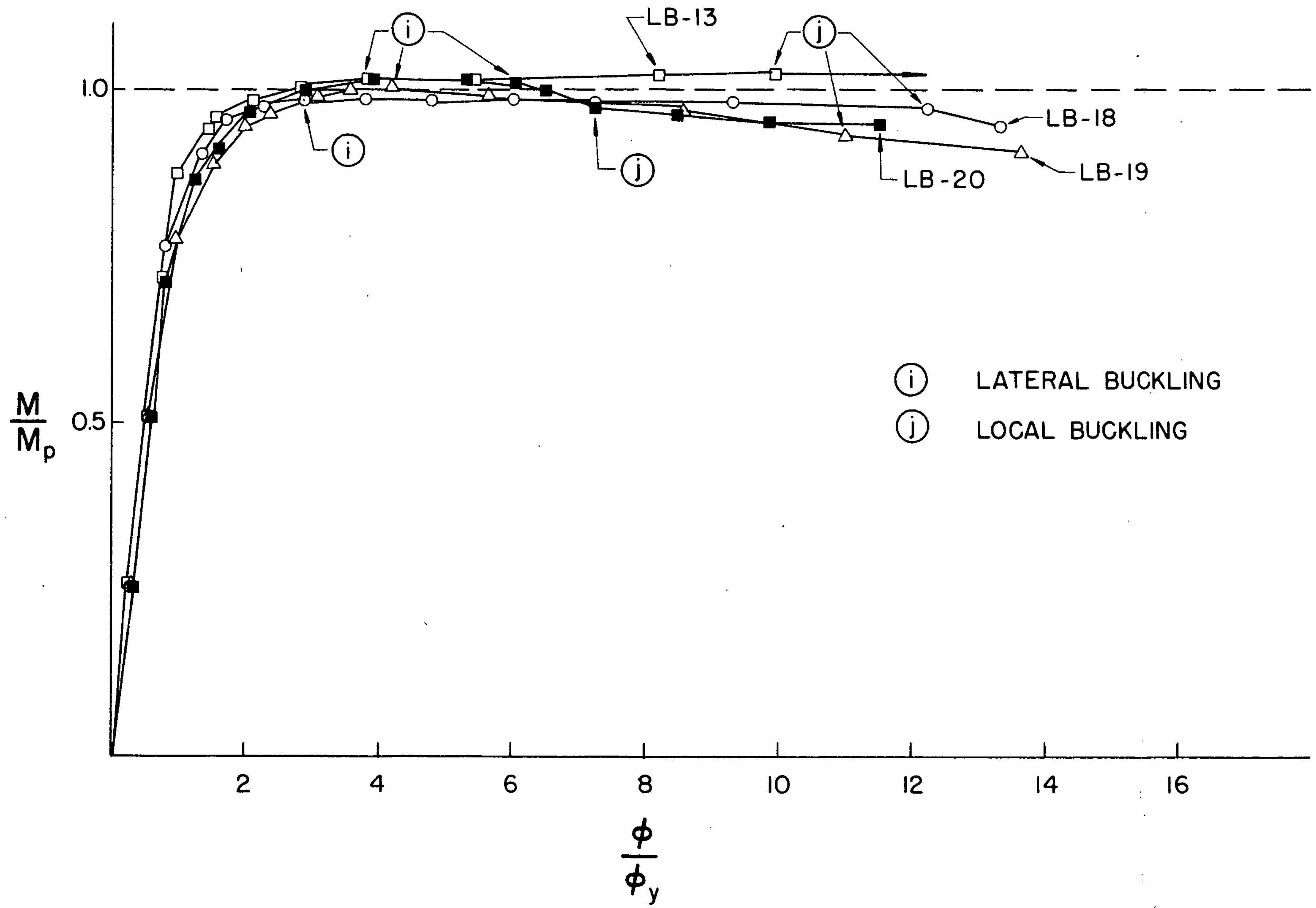


FIG. 13 MOMENT VERSUS MIDSPAN CURVATURE RELATIONSHIPS FOR LB-18, 19, 20 AND 13

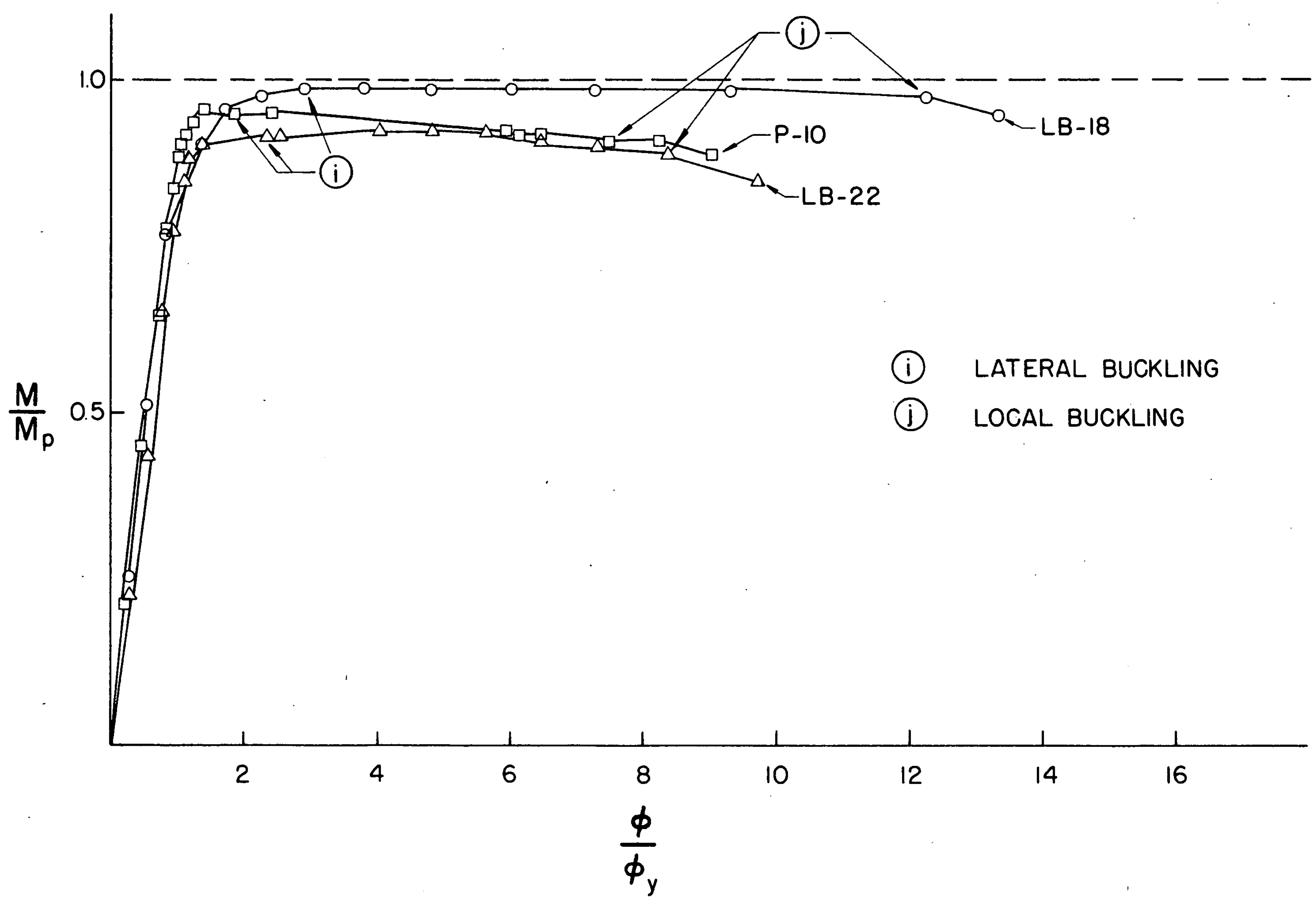


FIG. 14 MOMENT VERSUS MIDSPAN CURVATURE RELATIONSHIPS FOR LB-22, P-10, AND LB-18

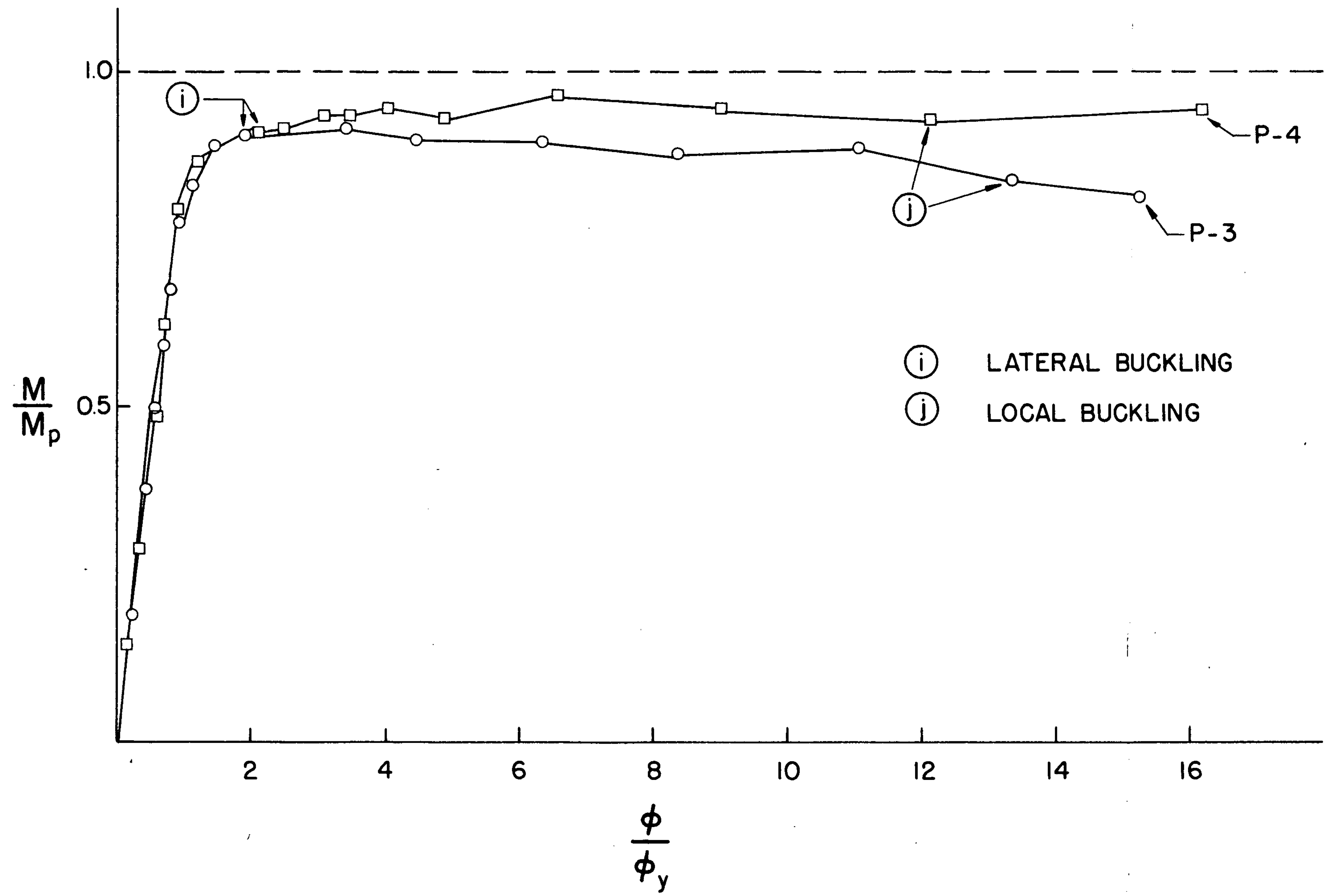


FIG. 15 MOMENT VERSUS MIDSPAN CURVATURE RELATIONSHIPS FOR P-3 AND P-4

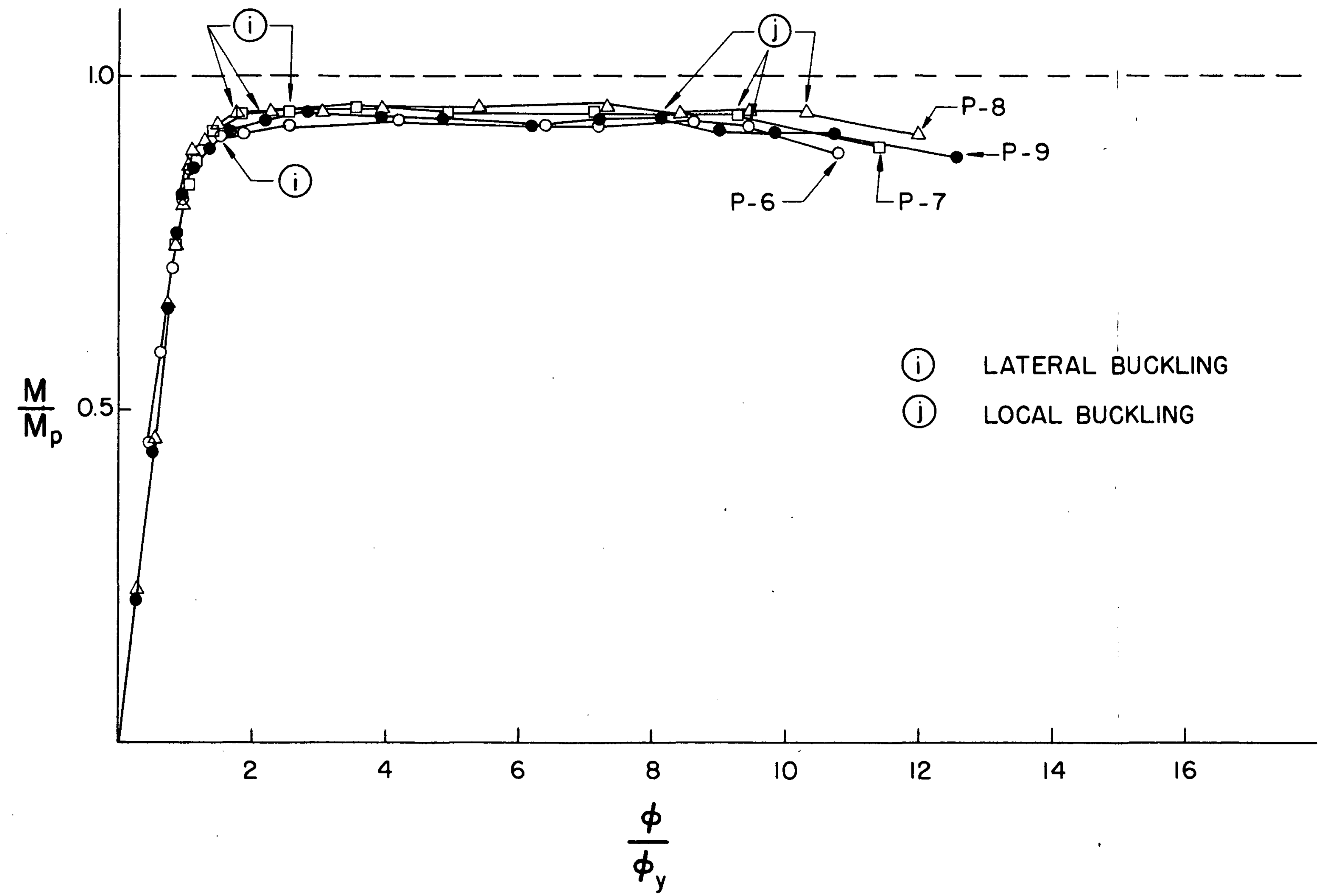


FIG. 16 MOMENT VERSUS MIDSPAN CURVATURE RELATIONSHIPS FOR P-6, 7, 8, AND 9

TEST NO.	M/M_p MAXIMUM			ϕ/ϕ_y @FAILURE			STRAIN†	LATERAL† DEFLECTION	VERTICAL† DEFLECTIONS		FAILURE BY
	(1)*	(2)	(3)	(1)*	(2)	(3)	(in./in.)	@ center (in.)	@ center↑	@ ends↓	
LB-12	1.02	0.96	1.10	20.82	19.72	22.40	<.026	1.37	- 1.4	4.9	Local buckling
LB-13	1.02	0.96	1.10	9.94	9.42	10.68	.017	.28	- 0.8	3.4	"
LB-14	1.00	0.95	1.08	7.33	6.94	7.88	.029	1.38	- 0.9	3.5	Inadequate bracing
LB-18	0.99	0.94	1.07	12.25	11.61	13.17	.014	0.84	- 0.6	3.3	Local buckling
LB-19	1.00	0.95	1.08	8.57	8.12	9.21	.014	0.91	- 0.6	?	Inadequate bracing
LB-20	1.02	0.96	1.10	7.23	6.85	7.78	.013	0.16	- 0.5	2.7	Inadequate bracing and local buckling
LB-22	0.92	0.98	1.11	8.35	8.86	10.10	.019	1.03	-0.6	3.1	"
P-3	0.91	0.93	1.15	13.32	13.63	16.89	<.026	0.80	- 0.5	2.3	Local buckling
P-4	0.96	0.98	1.22	12.11	12.40	15.35	<.023	0.47	-0.3	2.7	"
P-6	0.93	0.99	1.13	9.45	10.03	11.42	.017	1.01	-0.6	2.7	"
P-7	0.95	1.01	1.15	9.28	9.85	11.22	?	0.83	-0.3	2.8	"
P-8	0.96	1.02	1.16	10.30	10.94	12.47	.024	1.16	-0.8	3.3	"
P-9	0.95	1.01	1.15	8.12	8.62	9.82	.021	0.85	-0.56	2.7	"
P-10	0.96	1.02	1.16	7.50	7.97	9.07	.020	0.84	- 0.4	2.6	Inadequate bracing and local buckling

(1) using σ_y obtained from coupon tests
(2) using σ_y obtained from short beam test
(3) using nominal value of $\sigma_y = 33$ ksi

* – values used in presentation of results
† – @ local buckling

TABLE 4 TEST RESULTS

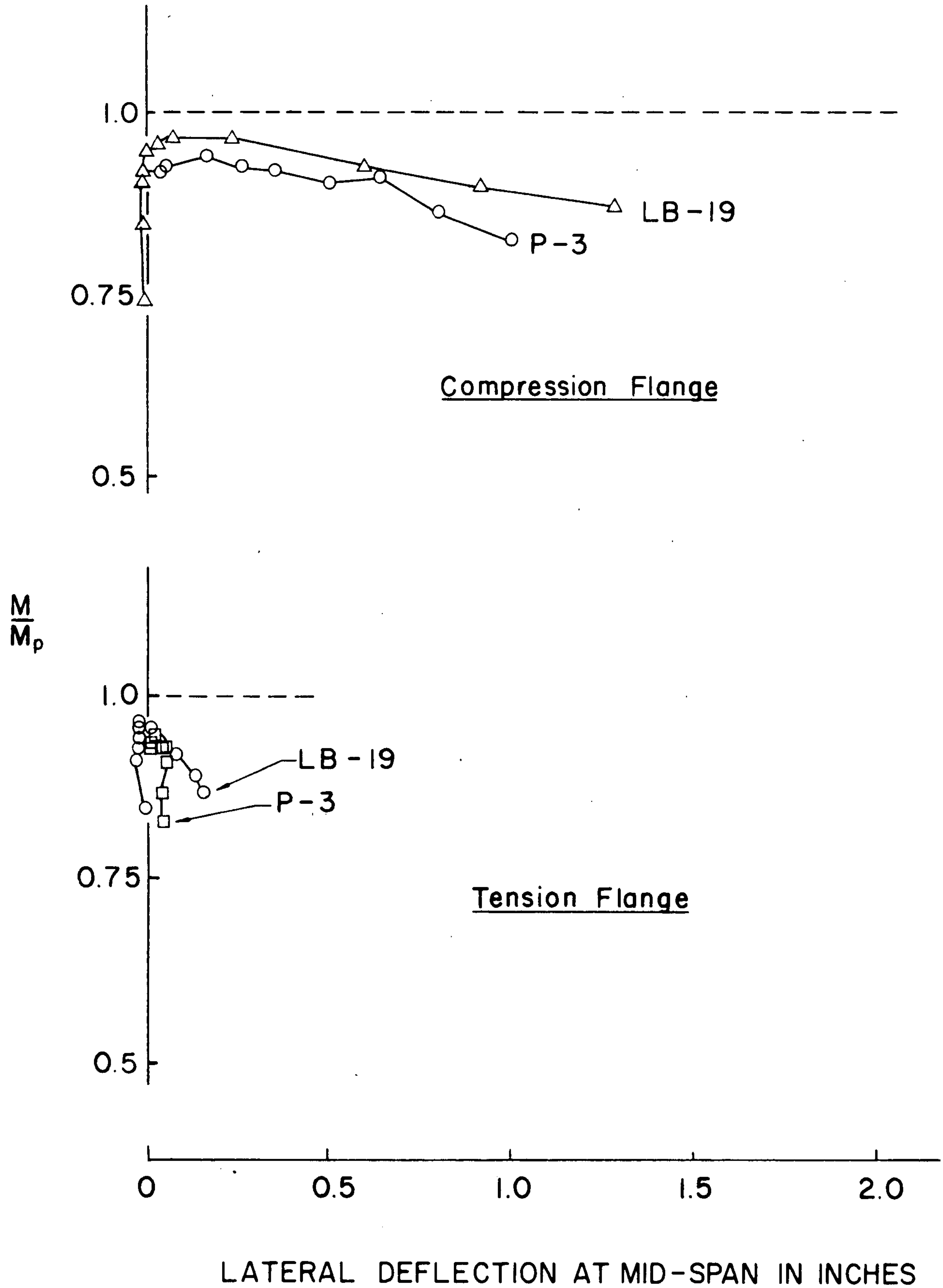


FIG. 17 MOMENT VERSUS LATERAL DEFLECTION AT MIDSPAN

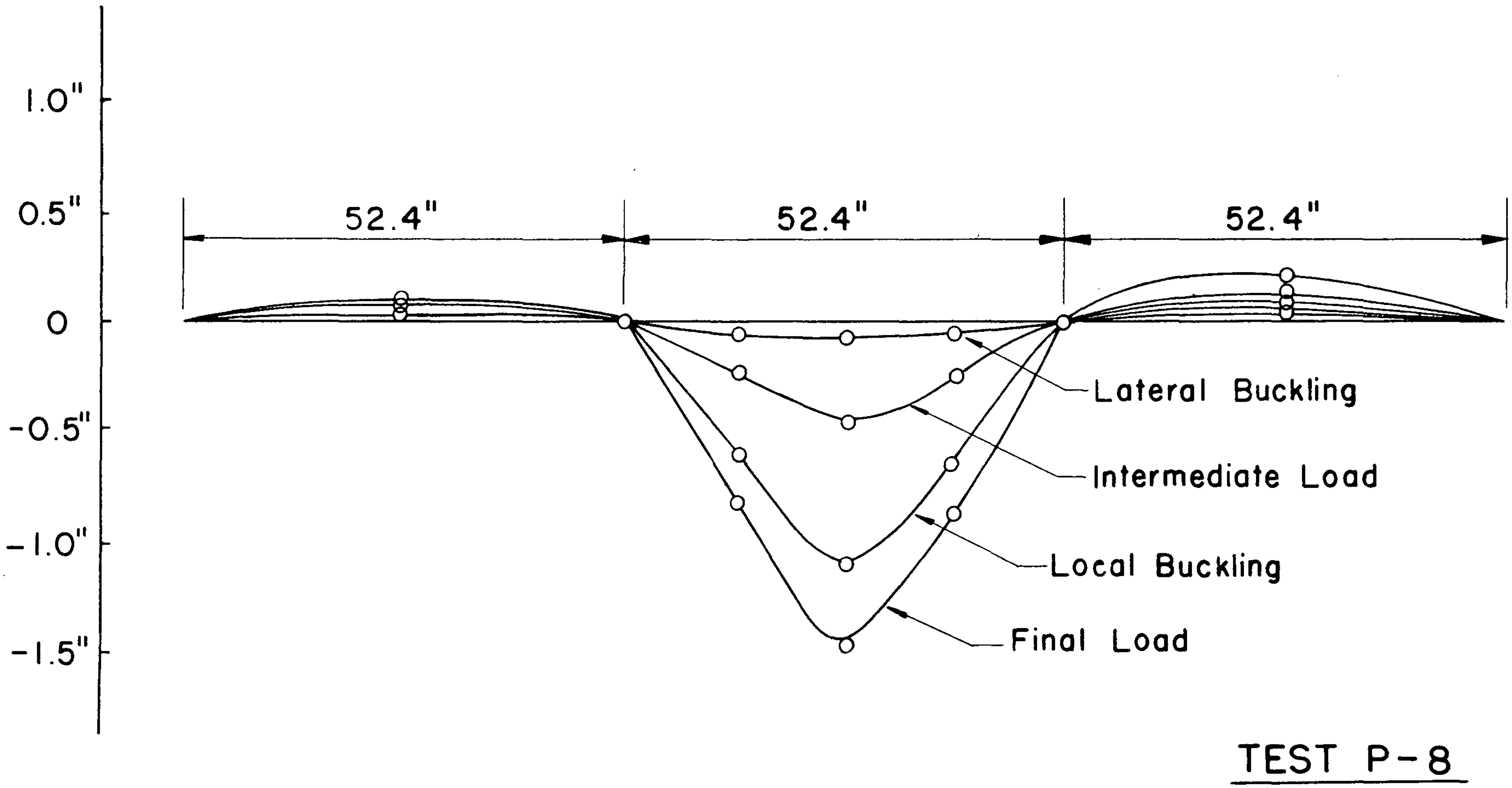


FIG. 18 LATERALLY DEFLECTED SHAPE OF A TYPICAL BEAM

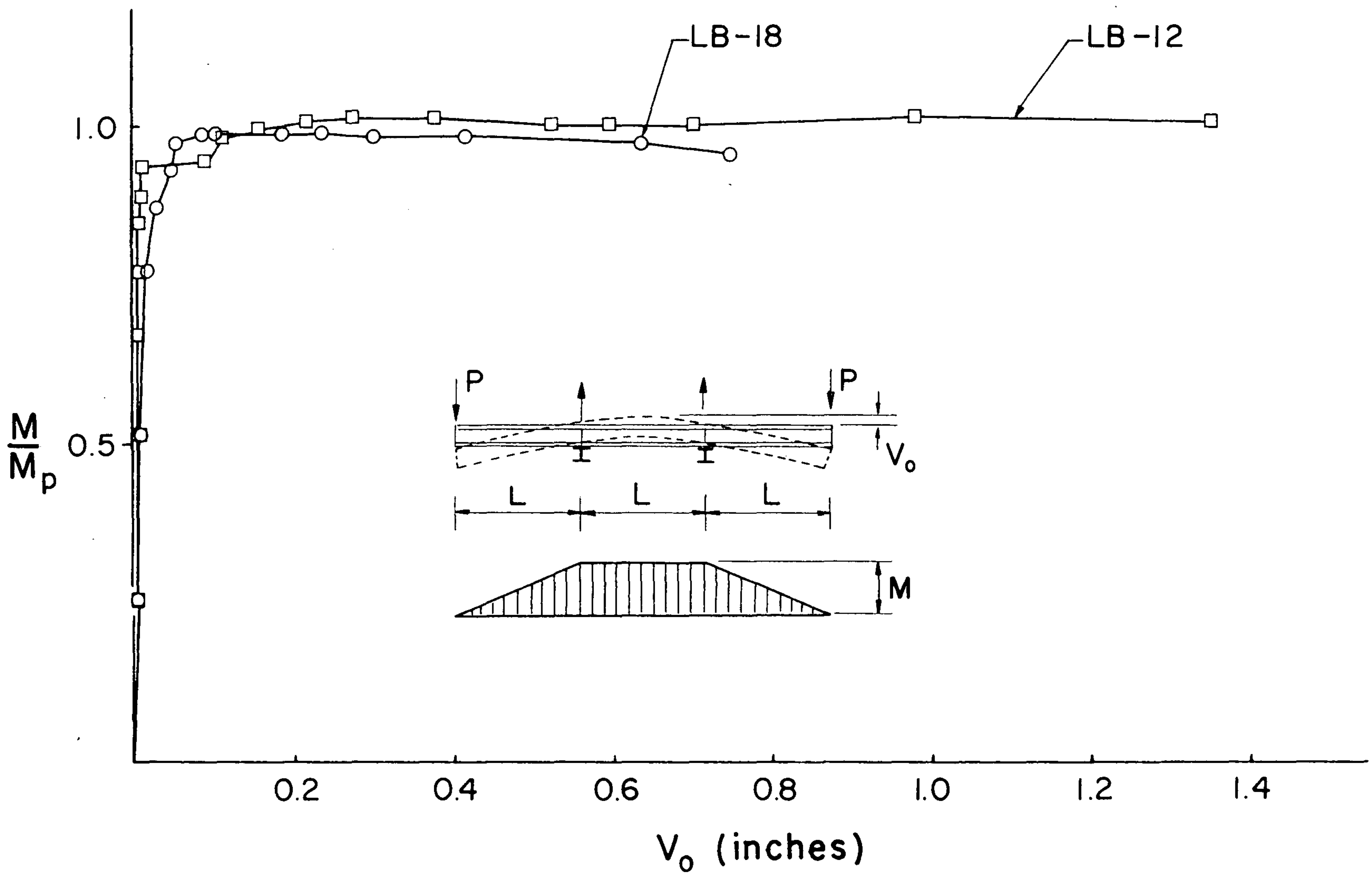


FIG. 19 MOMENT VERSUS VERTICAL DEFLECTION RELATIONSHIPS AT MID-SPAN

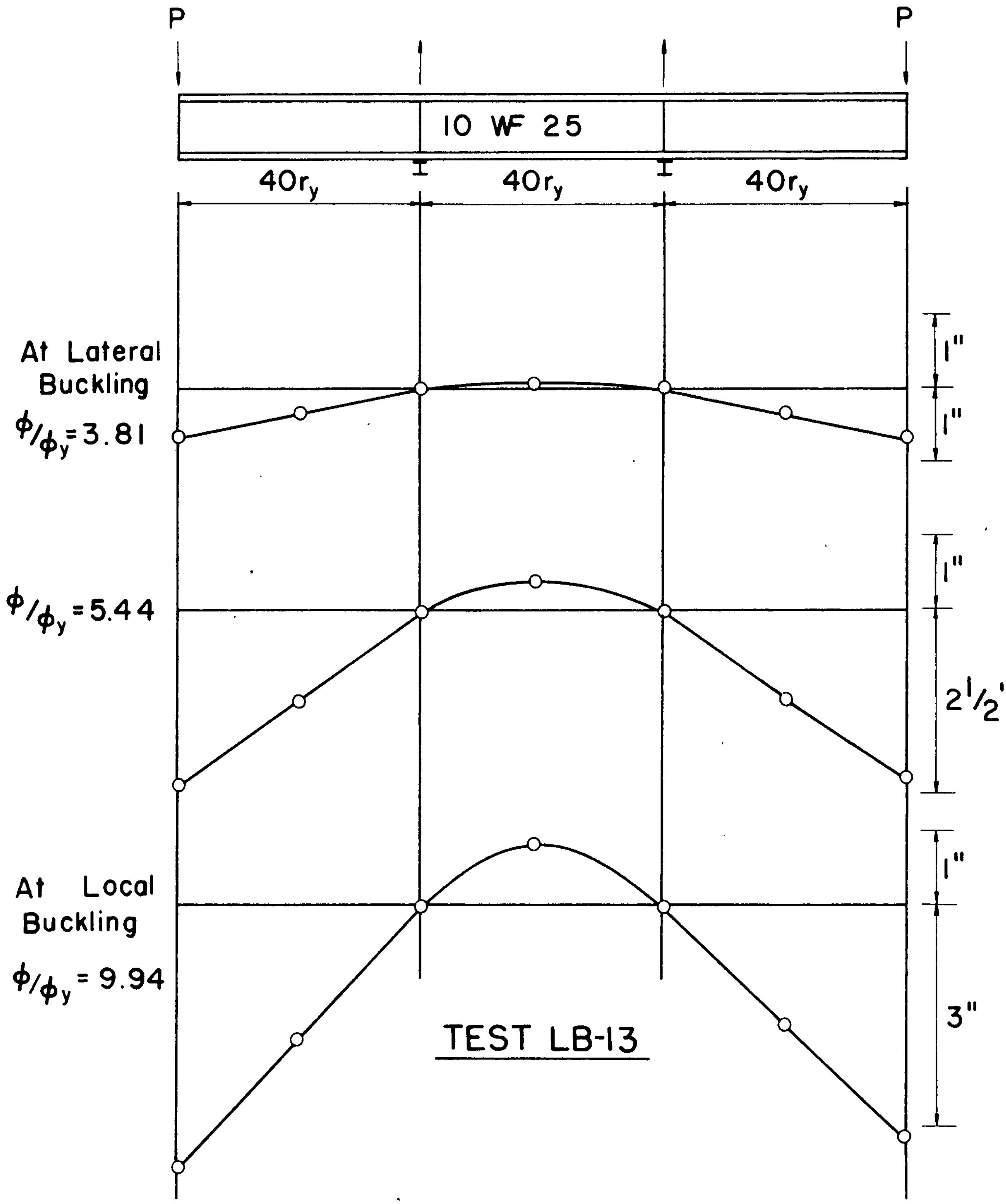


FIG. 20 VERTICAL DEFLECTIONS AT LATERAL BUCKLING, LOCAL BUCKLING, AND AN INTERMEDIATE POINT.

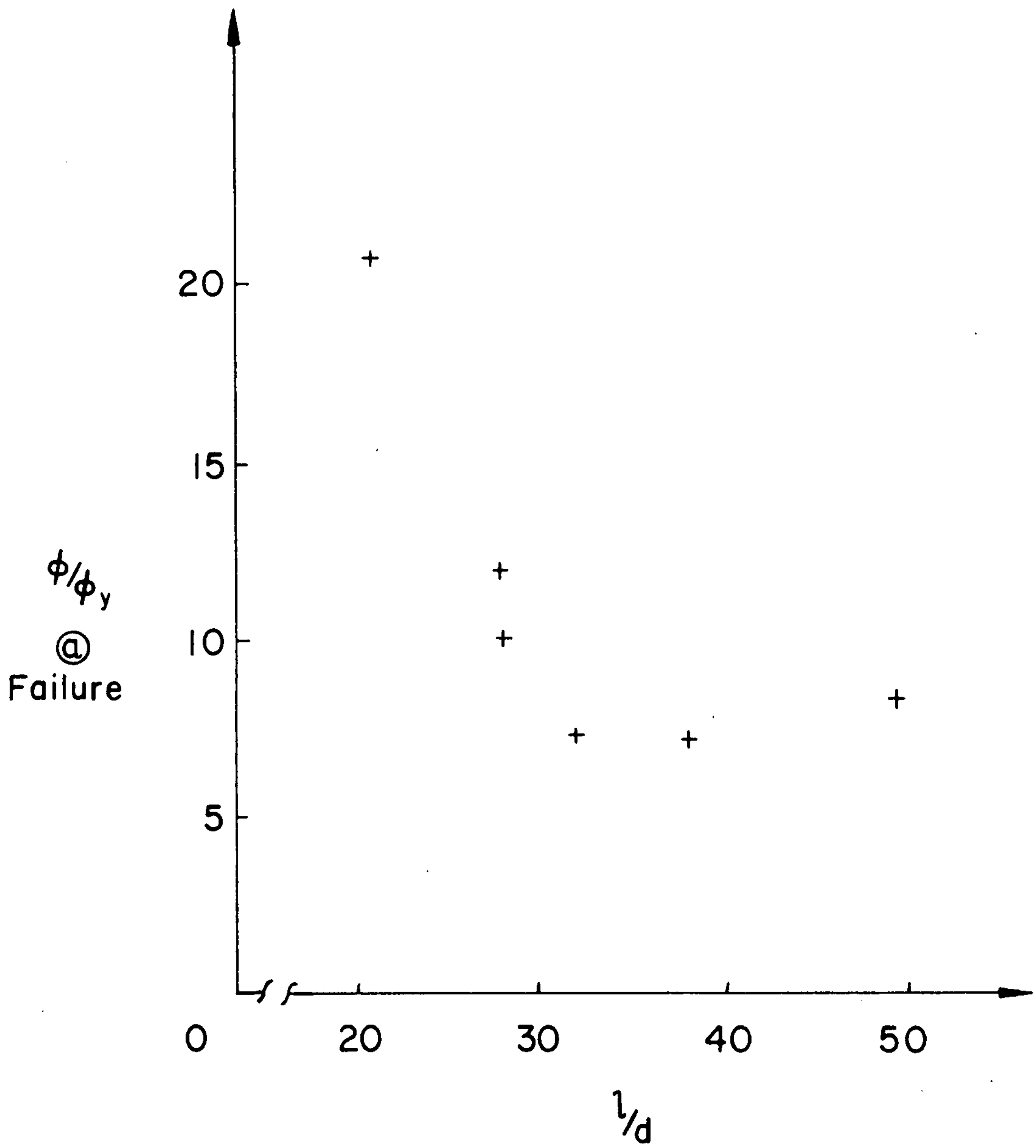


FIG. 21 CURVATURE AT FAILURE VERSUS l/d OF PURLINS FOR TESTS LB-12, 13, 14, 18, 19, 20.

8. VITA

The author was born in New York City, New York on December 1, 1939, the first child of Mr. and Mrs. Domenico T. Ferrara.

He was graduated from Stuyvesant High School in New York City, in June 1956. He then attended Manhattan College in Riverdale, New York from September, 1956 to June 1960. There he received the degree of Bachelor of Civil Engineering in September 1960.

He was awarded a Research Assistantship in Civil Engineering at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania and began studies for a Master's Degree there in September, 1960. The author has been associated with the research concerning lateral bracing requirements in plastic design.